Live Heap Space Analysis for Languages with Garbage Collection

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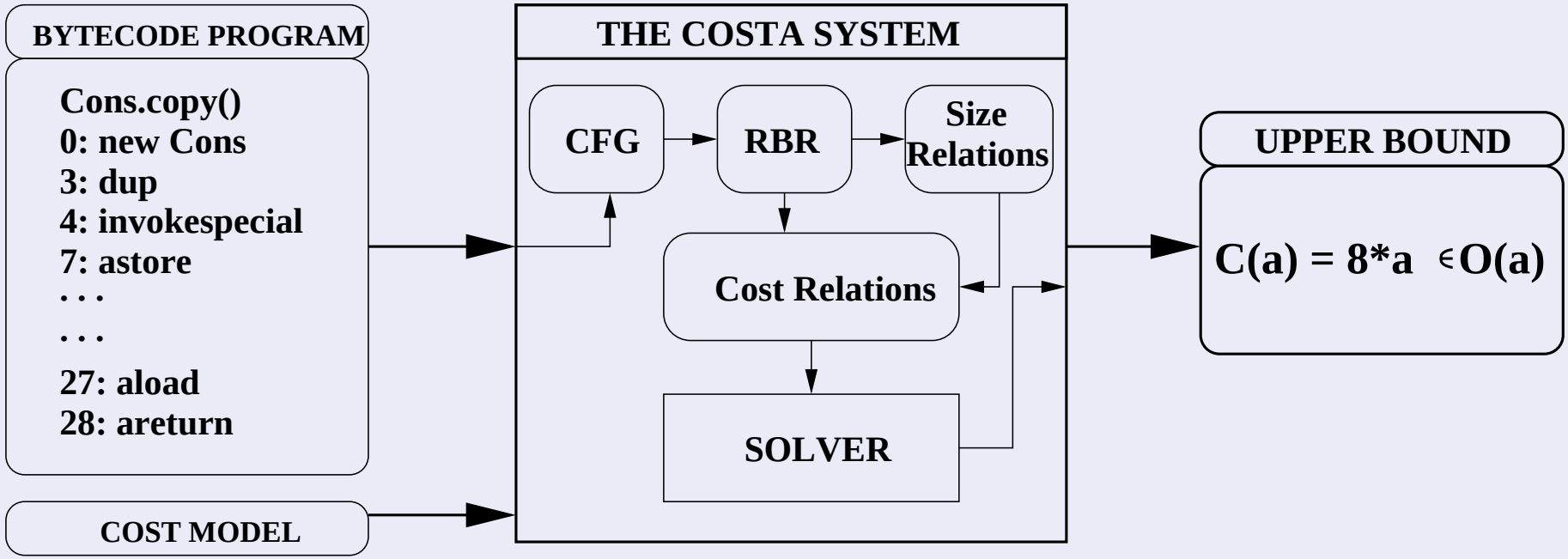
  - **Upper bounds**: a program runs within the resources available.
  - *Lower bounds*: useful for scheduling distributed execution.
- Two classes of upper bounds can be considered:
  - **non-asymptotic** (or concrete, or micro-analysis)
  - asymptotic (or macro-analysis)

COSTA: Cost Analyzer for Java Bytecode



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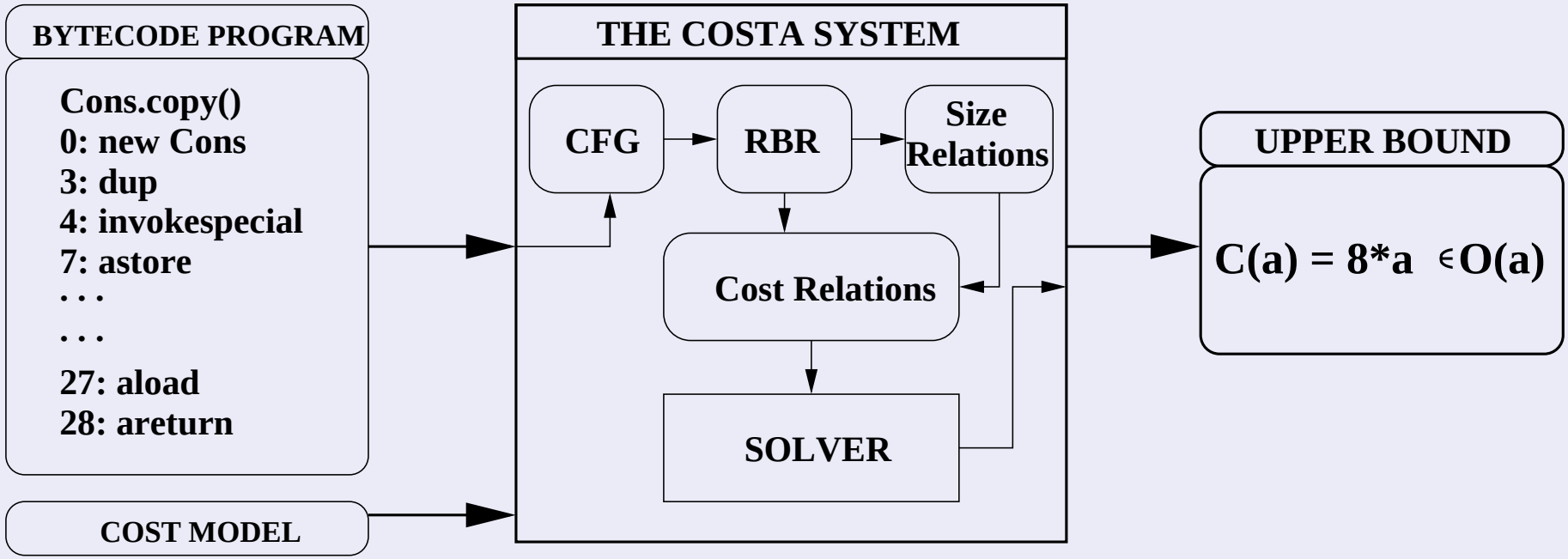

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- Java Bytecode: widely used (mobile systems), platform indep., etc

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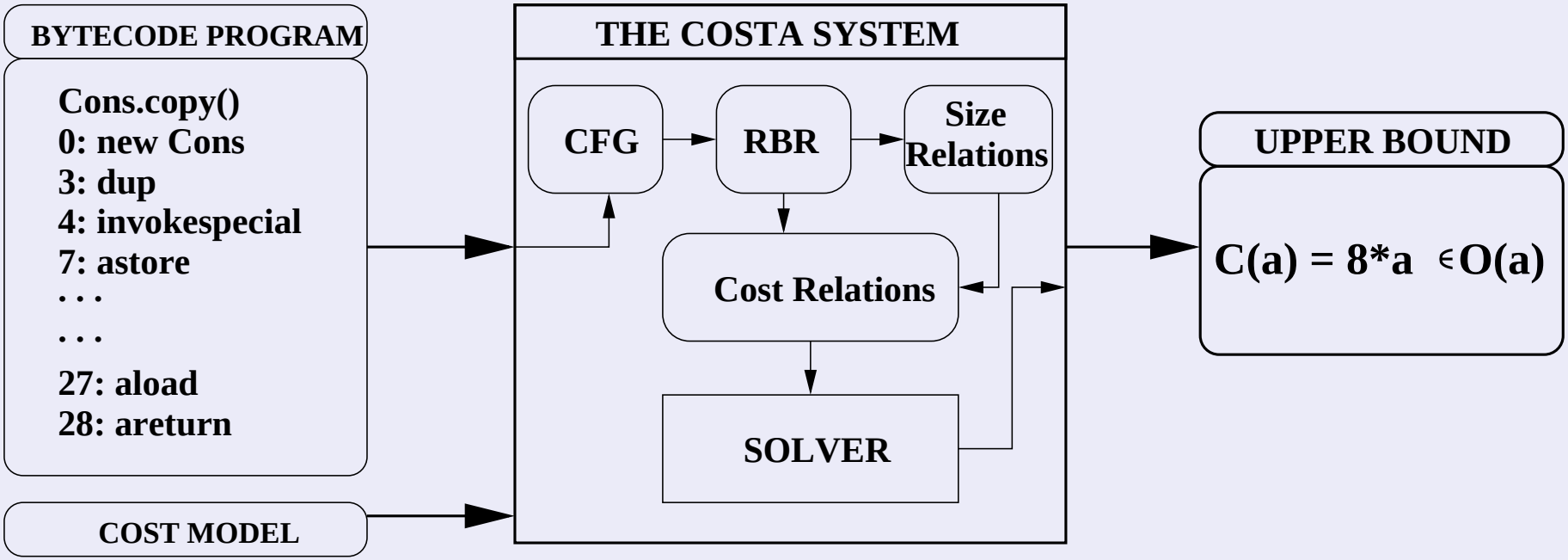

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I will focus on the main components and on cost models *heap* and *peak*.

HEAP: Total Allocation Analysis

- Symbolic *cost model* for heap consumption, *size*(Tree), *size*(Integer)..
- We get upper bounds of the *total* allocated memory.
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PEAK: Live Heap Space Analysis

- Aims at approximating the maximum or **peak** heap usage.
- Scope-based GC model:
  1. Reclaims unreachable memory when methods return. This assumption can be refined up to an ideal GC.
  2. Collects unreachable objects which have been created during the execution of the corresponding method call and not before.
- Much tighter estimation in presence of GC.

From Java to Intermediate representation

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class Test {
  static Tree m(int n) {
    if ( n>0 ) return
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  static int f(int n) {
    int a=0,i=n;
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$m(\langle n\rangle, \langle r\rangle) ::=$
$\quad \mathbf{n > 0},$
$\quad s_0 := \text{new } \text{Tree}^1;$
$\quad s_1 := n - 1,$
$\quad m(\langle s_1\rangle, \langle s_1\rangle),$
$\quad s_2 := n - 1,$
$\quad m(\langle s_2\rangle, \langle s_2\rangle),$
$\quad f(\langle n\rangle, \langle s_3\rangle),$
$\quad init(\langle s_0, s_1, s_2, s_3\rangle, \langle\rangle),$
$\quad r = s_0.$

$m(\langle n\rangle, \langle r\rangle) ::=$
$\quad \mathbf{n \leq 0},$
$\quad r := \text{null}.$

$$m(n) = \text{size}(\text{Tree}^1) + m(n-1) + m(n-1) + f(n) + init(1, n-1, n-1, s_3) \quad \{n>0\}$$

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$$
\begin{array}{lll}
m(n) = \mathrm{size}(\mathrm{Tree}^1) + & & \{n>0\} \\
\qquad m(n-1) + m(n-1) + f(n) + & & \\
\qquad \mathrm{init}(1, n-1, n-1, s_3) & & \\
m(n) = 0 & & \{n \leq 0\} \\
f(n) = f_c(n, 0) + f_d(n, a') & & \{\} \\
f_c(n, a) = g(n) + f_c(n/2, a') & & \{n>1\} \\
f_c(n, a) = 0 & & \{n \leq 1\} \\
f_d(i, a) = h(i) + f_d(i/2, a') & & \{i>1\} \\
f_d(i, a) = 0 & & \{i \leq 0\} \\
g(n) = \mathrm{size}(\mathrm{Integer}^2) + \mathrm{size}(\mathrm{Integer}^3) & & \{\} \\
h(n) = \mathrm{size}(\mathrm{Long}^4) & & \{\} \\
\mathrm{init}(\mathrm{this}, l, r, d) = 0 & & \{\}
\end{array}
$$

From Cost Relations to Upper Bounds

solutions computed for f

$total(f) = total(f_c) + total(f_d)$

```
for (; n>1 ; n=n/2 ) a += g(n).intValue();
for (; i>1 ; i=i/2 ) a *= h(i).intValue();
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$f_c(n) = log(n) * (size(\mathrm{Integer}^3) + size(\mathrm{Integer}^2))$
$f_d(i) = log(i) * size(\mathrm{Long}^4)$

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solutions computed for m

$m(n) = 2^n * (\underbrace{total(f)}_{exp\ \ times} + size(\mathrm{Tree}^1))$

```
new Tree(m(n-1),m(n-1),f(n));
```

Why Live Heap Space Analysis is Different?

Basic idea in total allocation:

$$\mathbf{total}(\{\mathbf{m_1}; \mathbf{m_2}\}) = \mathbf{total}(\mathbf{m_1}) + \mathbf{total}(\mathbf{m_2})$$

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$$\mathbf{peak}(\{\mathbf{m_1}; \mathbf{m_2}\}) = \mathbf{max}(\mathbf{peak}(\mathbf{m_1}), \mathbf{escaped}(\mathbf{m_1}) + \mathbf{peak}(\mathbf{m_2}))$$

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1. STEP 1: Escaped memory analysis
2. STEP 2: Peak consumption cost relations

Example

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  static Tree m(int n) {
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    else return null;
  }

  static int f(int n) {
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    for(; i>1; i=i/2)
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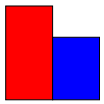
g peak
g escaped



1 2

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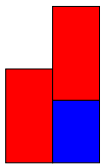

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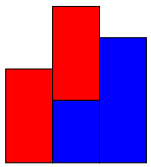

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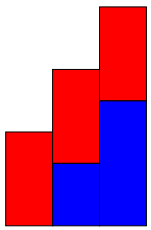

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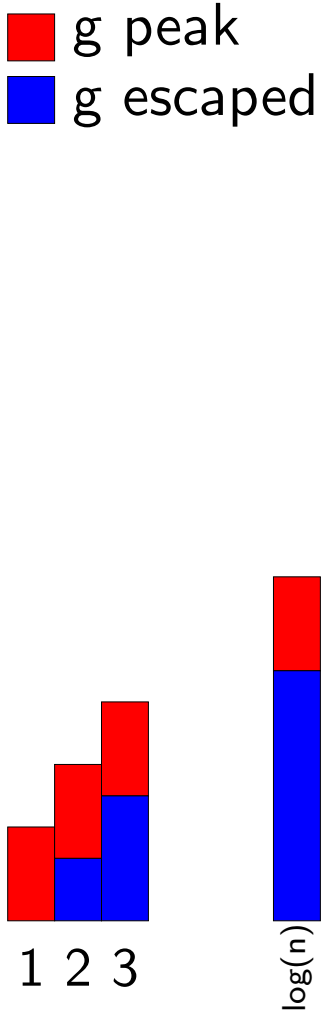

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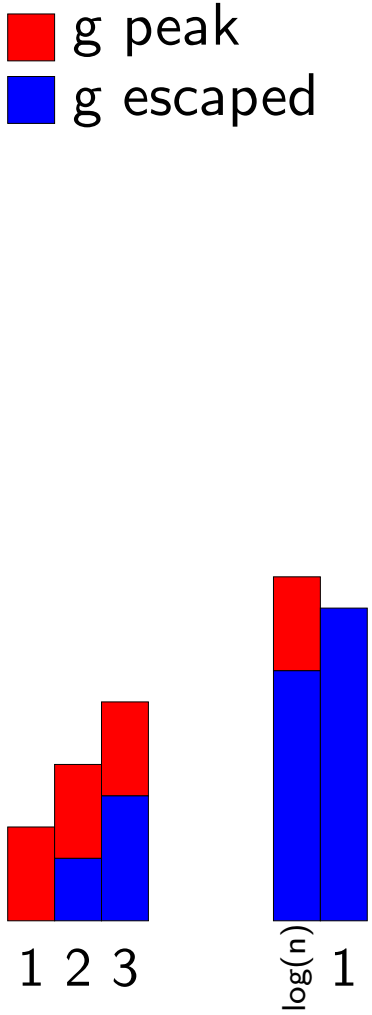

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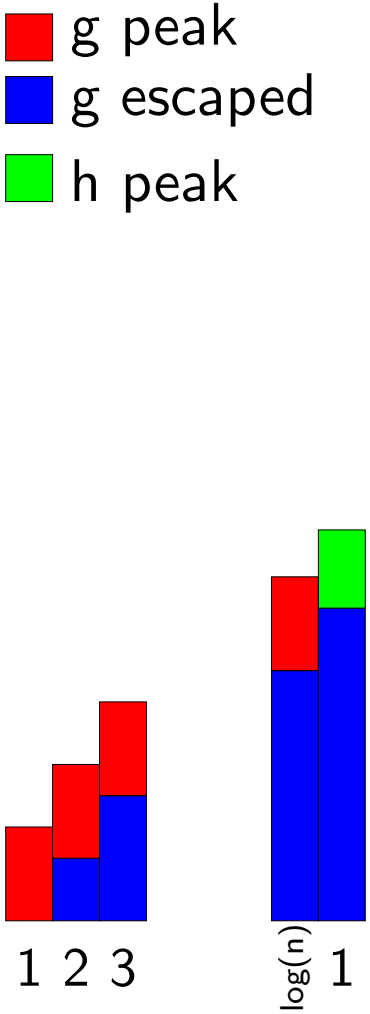

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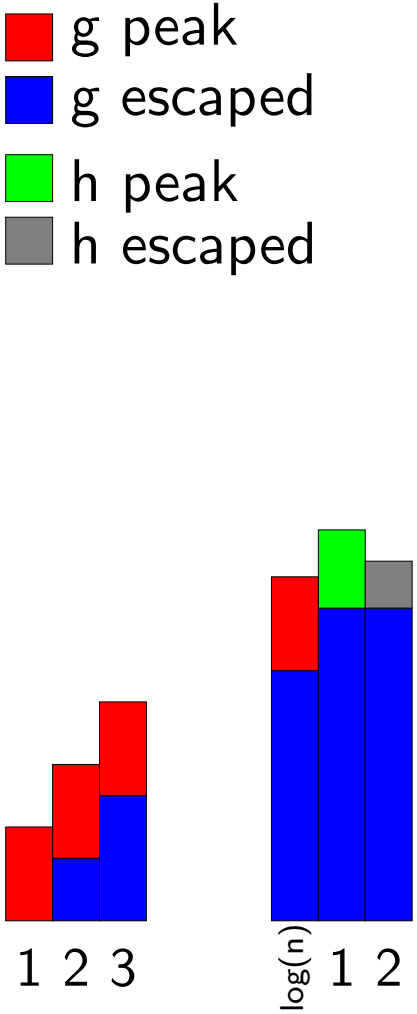

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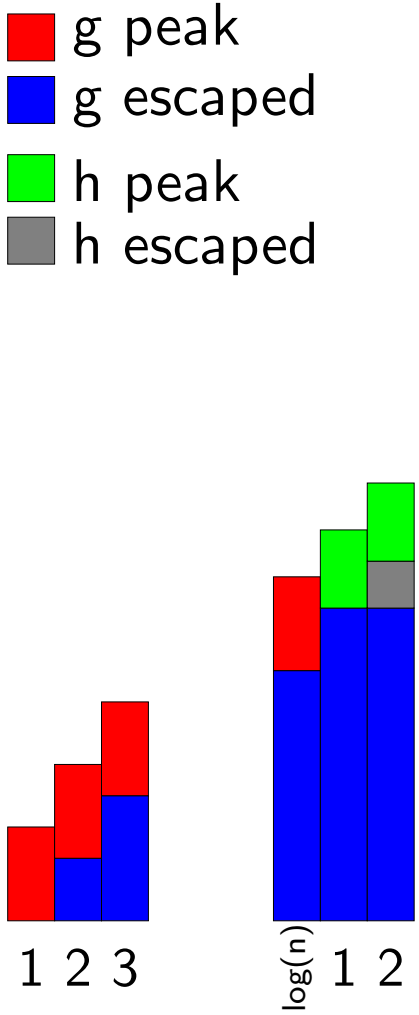

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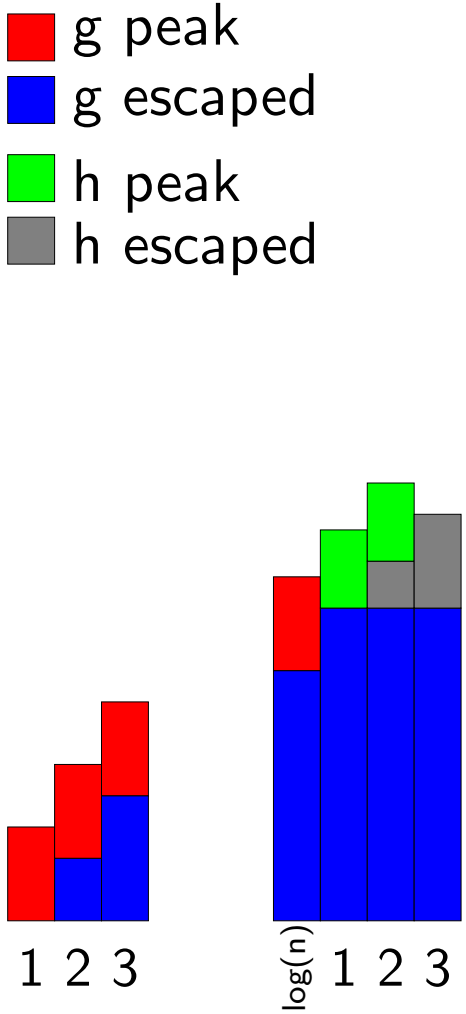

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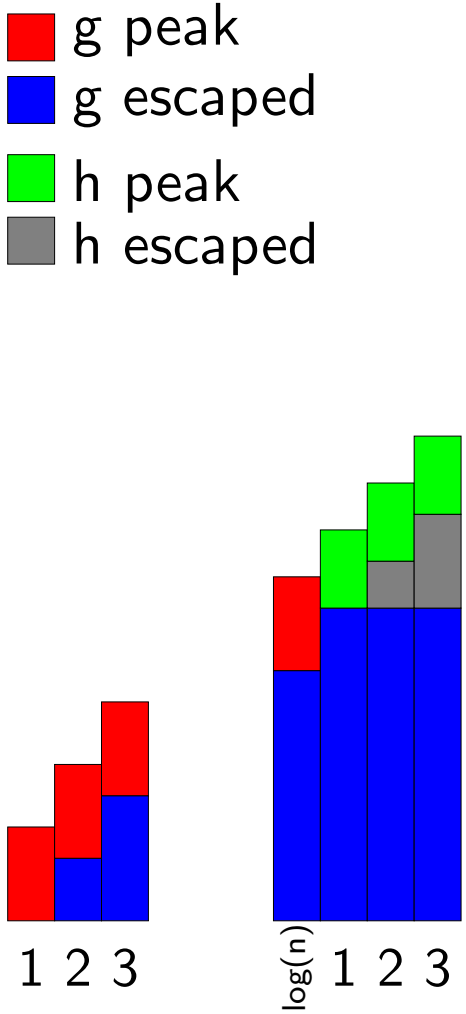

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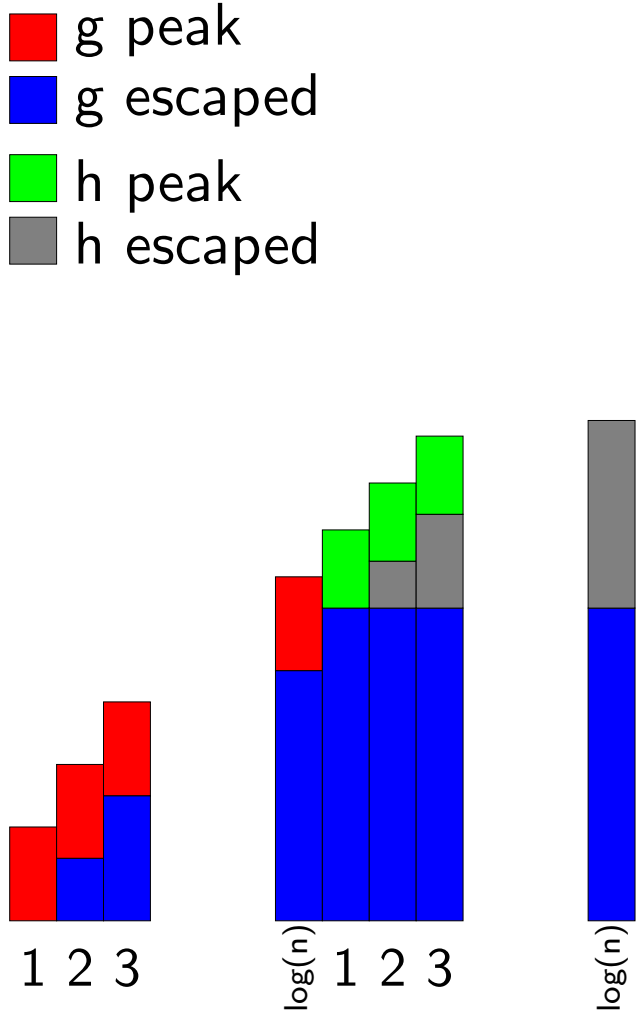

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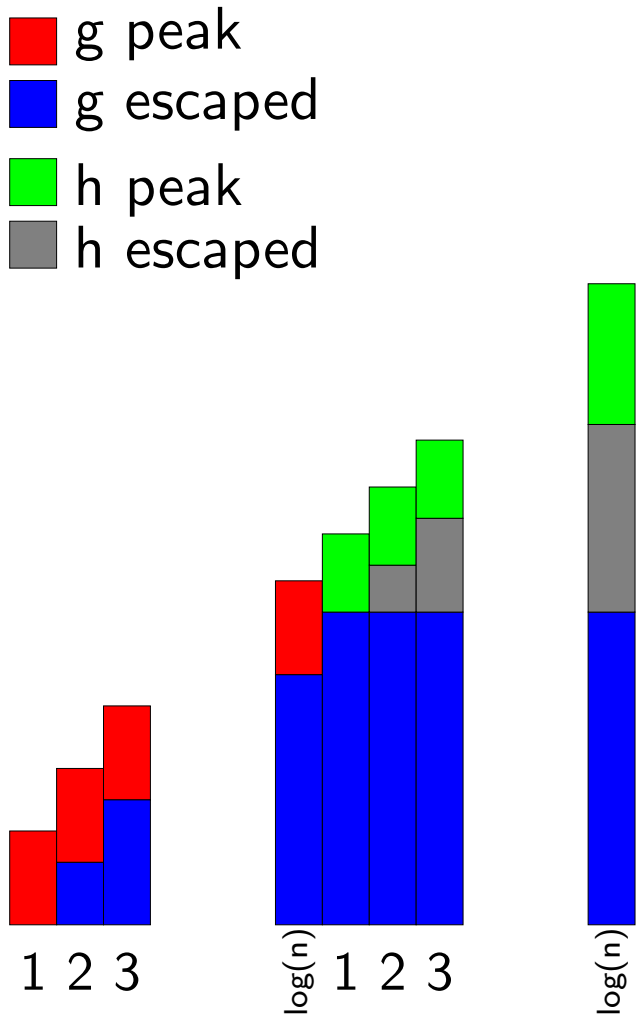

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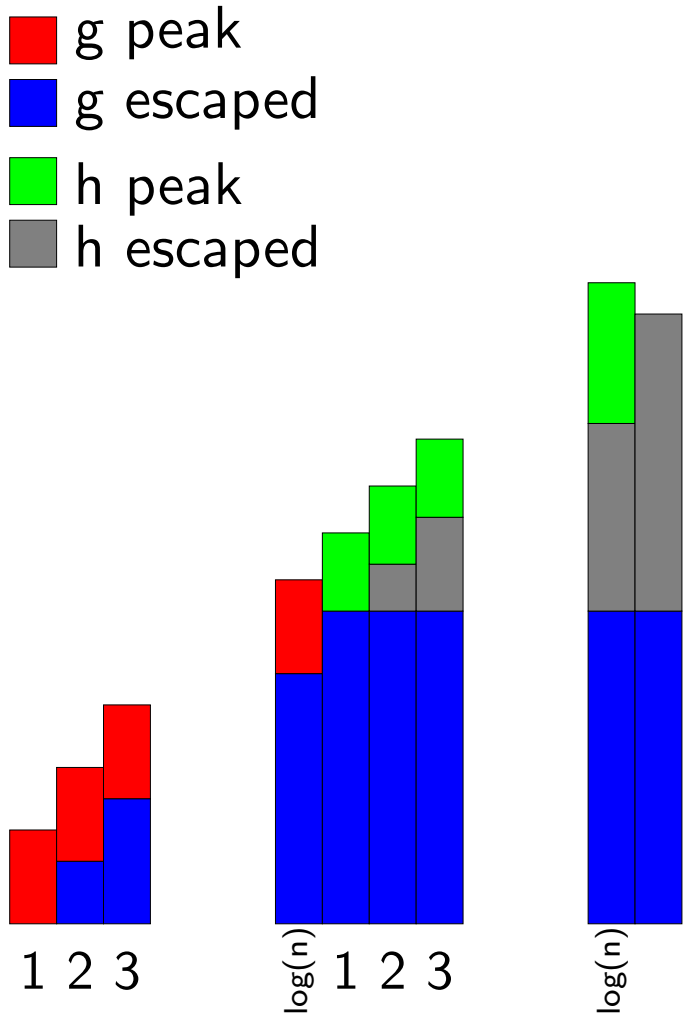

Step 1: Inference of Escaped Memory Bounds

- Infer upper bounds on the total memory consumption
- Remove from the bounds the collectable objects
- The set of collectable objects can be approximated from the information computed by escape analysis

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- Remove from the bounds the collectable objects
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collectable objects from m

the objects that have been created along the execution of m and will not be in the memory upon exit from m

Collectable Objects

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class Test {
  static Tree m(int n) {
    if ( n>0 ) return new
      Tree(m(n-1),m(n-1),f(n));
    else return null;
  }

  static int f(int n) {
    int a=0,i=n;
    for(; n>1;n=n/2 )
      a += g(n).intValue();
    for(; i>1; i=i/2)
      a *= h(i).intValue();
    return a;
  }

  static Integer g(int n) {
    Integer x=new Integer^2(n);
    return new Integer^3(x.intValue()+1);
  }

  static Long h(int n) {
    return new Long^4(n-1);
  }
} // end of class Test
```

$$\mathit{collectable}(\mathrm{g}) = \{\mathrm{Integer}^2\}$$
$$\mathit{collectable}(\mathrm{h}) = \emptyset$$
$$\mathit{collectable}(\mathrm{f}) = \{\mathrm{Integer}^2, \mathrm{Integer}^3, \mathrm{Long}^4\}$$
$$\mathit{collectable}(\mathrm{m}) = \{\mathrm{Integer}^2, \mathrm{Integer}^3, \mathrm{Long}^4\}$$

Collectable Objects

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      a *= h(i).intValue();
    return a;
  }
```

-
```
  static Integer g(int n) {
    Integer x=new Integer²(n);
    return new Integer³(x.intValue()+1);
  }

  static Long h(int n) {
    return new Long⁴(n-1);
  }
} // end of class Test
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Collectable Objects

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Bounds on Escaped Memory

escaped memory of a procedure p

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Step 2: Inference of Live Memory Bounds

- Build cost relations which capture the basic idea

$$\mathbf{peak}(\{\mathbf{m_1};\mathbf{m_2}\}) = \mathbf{max}(\mathbf{peak}(\mathbf{m_1}), \mathbf{escaped}(\mathbf{m_1}) + \mathbf{peak}(\mathbf{m_2}))$$

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peak cost relation

$\mathcal{T}(b_1,\ldots,b_n)::=$

- if b_1 is a call, then $max(peak(b_1), escaped(b_1) + \mathcal{T}(b_2,\ldots,b_n))$
- if b_1 is an instruction, then $total(b_1) + \mathcal{T}(b_2,\ldots,b_n)$

Detailed Example

- For the following code in m:

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new Tree(m(n-1),m(n-1),f(n));
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- An upper bound of the equations:

$$\boxed{peak_m(n) = 2^n * size(\mathrm{Tree}^1) + peak_f(n)}$$

Live Heap Bounds vs. Total Memory Bounds

solutions computed from peak cost relation

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tighter bound for *f*

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Conclusions

- We have presented an automatic live heap space analysis for garbage-collected languages.
- It generates at compile-time cost relations which define the peak consumption of a program as a function of its input data size.
- The CRs obtained can be solved with standard UBs solvers (PUBS).
- We have successfully analyzed the JOlden benchmark suite.
- Applications: verification, certification (embedded systems, critical/real-time apps), program optimization and understanding, etc.

Future Work

- Adapt our techniques to region-based garbage collection
- The idea could be used to estimate other non-accumulative resources.

Info on Costa:

`http://costa.ls.fi.upm.es`