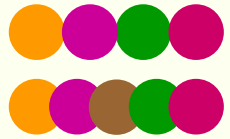


Structural Contradictions

Cindy Eisner

IBM Haifa Research Laboratory

Joint work with Dana Fisman



Structural contradiction

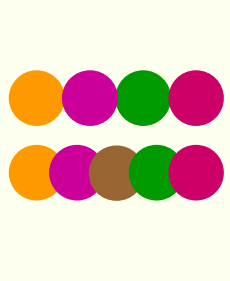
- A SERE[‡] not satisfiable due to its structure

$$p \cap (p \cdot p)$$

- Contrast with logical contradiction
 - Propositional formula that is not satisfiable

$$p \wedge \neg p$$

[‡] Semi-extended regular expression



The structure

- Logical contradiction
 - Exists valid proposition of same structure

$p \wedge \neg p$



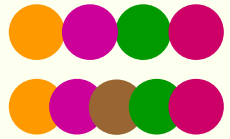
$\text{true} \wedge \neg \text{false}$

- Structural contradiction
 - No SERE of same structure is satisfiable

$p \cap (p \cdot p)$



$\text{true} \cap (\text{true} \cdot \text{true})$



More on structure

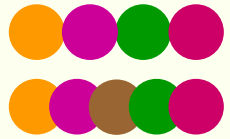
- For a propositional formula
 - One kind of contradiction (logical)

$$p \wedge \neg p$$

- For a SERE
 - Two kinds (structural and non-structural)

$$p \cap (p \cdot p)$$

$$(p \cdot p) \cap (p \cdot \neg p)$$



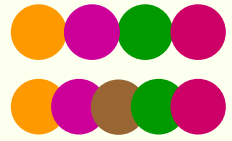
\cap is not the only source

- The fusion operator \circ
 - A kind of overlapping concatenation
 - $\mathcal{L}(r_1 \circ r_2) = \{ulv \mid ul \in \mathcal{L}(r_1) \text{ and } lv \in \mathcal{L}(r_2)\}$

$\langle a \rangle \langle b \rangle \langle cd \rangle \langle e \rangle \langle f \rangle$

$\in \mathcal{L}((a \cdot b \cdot c) \circ (d \cdot e \cdot f))$

$\langle a \rangle$ is a letter such that atomic proposition a and only atomic proposition a holds

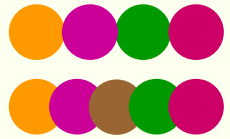


In social theory

- **Structural contradiction**
 - Contradiction that can only be resolved by dismantling existing structure of power*
- According to Marxist theory
 - **Capitalism** is a structural contradiction
 - Though Karl Marx viewed bourgeois society as more of a process than a structure†

* John Croft, history forum, amazon.com

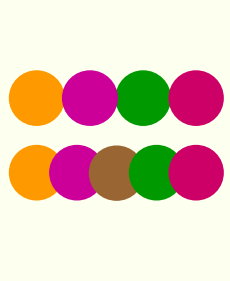
† Andy Blunden of marxists.org, personal communication



Back to temporal logic

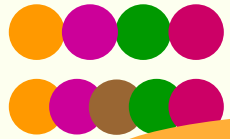
Why do we care?

- In Due to lack of time, we'll skip the truncated semantics and show only the issue of $r!$ vs. r
- Re See paper for all the gory details on the truncated semantics
 - **P**
 - **bl**
- Similar issue in **truncated semantics**



Outline

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Semantics intuition

For every prefix in the language of r :

There exists a prefix in $\mathcal{L}(r)$:

Concatenation: $\langle a \rangle \langle b \rangle \langle c \rangle \in \mathcal{L}(r)$!

$\langle a \rangle \langle b \rangle \langle b \rangle \langle c \rangle \in \mathcal{L}((a \cdot b) \cdot (b \cdot c))$ but that:

• $\langle a \rangle \langle b \rangle \langle b \rangle \langle c \rangle \in \mathcal{L}((a \cdot b) \circ (b \cdot c))$
 Overlapping concatenation:
 $\langle a \rangle \langle b \rangle \langle c \rangle \in \mathcal{L}((a \cdot b) \circ (b \cdot c))$

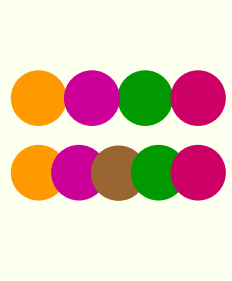
• or we may write $\langle a \rangle \langle b \rangle^{\omega} \models (a \cdot b^* \cdot c)$

Note similarity to weak until (W)

$r!$

r

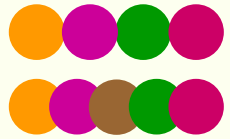
$r \mapsto \varphi$



The T, \perp approach

- Augment alphabet with two special letters
 - Anything holds on T (even false)
 - Nothing holds on \perp (even true)
- Define \bar{w} to be the word obtained by
 - Replacing every T with \perp and vice versa
- LTL operators
 - Standard, except that negation uses \bar{w}

$$w \models_{T\perp} \neg\varphi \iff \bar{w} \not\models_{T\perp} \varphi$$



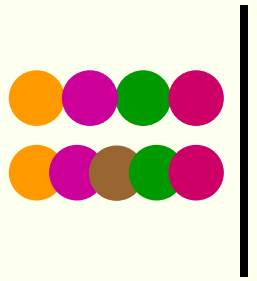
Semantics of T, \perp approach

- SERE formulas

$$w \models_{T\perp} r! \iff \exists j < |w| \text{ s.t. } w^{0..j} \in \mathcal{L}(r)$$

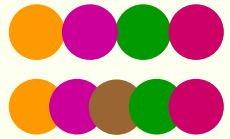
$$w \models_{T\perp} r \iff \forall j < |w|, w^{0..j} T^w \models_{T\perp} r!$$

$$w \models_{T\perp} r \mapsto \varphi \iff \\ \forall j < |w| \text{ s.t. } \overline{w}^{0..j} \in \mathcal{L}(r), w^{j..} \models_{T\perp} \varphi$$



Outline

- The semantics of SERE formulas
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What breaks

- Let $\varphi = (a \cdot b^* \cdot \text{false})$
 $\varphi' = (a \cdot b^* \cdot (c \cap (c \cdot c)))$

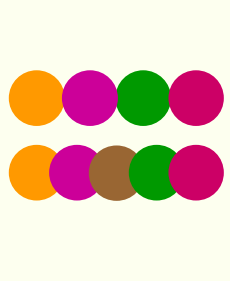
- Hopefully, it is easy to see that

$$\langle a \rangle \langle b \rangle \langle b \rangle \langle b \rangle \models_{T\perp} \varphi$$

$$\langle a \rangle \langle b \rangle \langle b \rangle \langle b \rangle \not\models_{T\perp} \varphi'$$

- T "catches" the false, but not $(c \cap (c \cdot c))$

$$w \models_{T\perp} r \iff \forall j < |w|, w^{0..j} T^\omega \models_{T\perp} r!$$



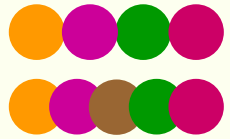
Outline

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- One fix that does

Why am I showing you these?

To show that it's not so easy to get it right.

(We tried and discarded these fixes.)



Bad fix 1: "flexible letters"

- Define
 - $\mathcal{L}(\lambda) = T^*$
 - $\mathcal{L}(b) = \{\ell \text{ s.t. } \ell \models b\} \cdot T^*$
- Now

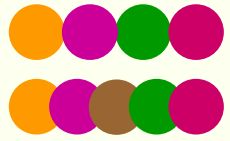
One T for false

$\langle a \rangle \langle b \rangle \langle b \rangle \langle b \rangle \models_{T\perp} (a \cdot b^* \cdot \text{false})$

$\langle a \rangle \langle b \rangle \langle b \rangle \langle b \rangle \models_{T\perp} (a \cdot b^* \cdot (c \cap (c \cdot c)))$

Two T's for c

And two for $(c \cdot c)$



So what's wrong

We've changed the semantics!

- Let $r_1 = (a \cdot b \cdot c)$, $r_2 = (d \cdot e \cdot f)$, $\varphi = r_1 \circ r_2$

Original semantics
(one T per atom)

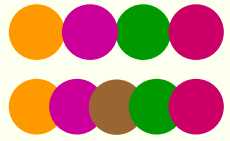
$\langle a \rangle \langle b \rangle \langle c \rangle \not\models_{T \perp} \varphi$

$\langle a \rangle \langle b \rangle \langle c \rangle$ "matches" r_1 ,
but $\langle c \rangle TT$ does not "match" r_2

Flexible letter
approach

$\langle a \rangle \langle b \rangle \langle c \rangle \models_{T \perp} \varphi$

we can use $\langle a \rangle \langle b \rangle \langle c \rangle T$ to "match"
 r_1 , and TTT to "match" r_2



Bad fix 2: weak language

"Too short"

"Too long"

- $\mathcal{L}_{\text{weak}}(\lambda) = \varepsilon$
 $\mathcal{L}_{\text{weak}}(b) = \varepsilon \cup \{\ell \text{ s.t. } \ell \models b\}$ SERE, include "t" or "too long"

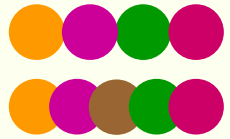
$$\mathcal{L}_{\text{weak}}(r_1 \cdot r_2) = \mathcal{L}_{\text{weak}}(r_1) \cup (\mathcal{L}(r_1) \cdot \mathcal{L}_{\text{weak}}(r_2))$$

$$\mathcal{L}_{\text{weak}}(r_1 \circ r_2) = \mathcal{L}_{\text{weak}}(r_1) \cup (\mathcal{L}(r_1) \circ \mathcal{L}_{\text{weak}}(r_2))$$

$$\mathcal{L}_{\text{weak}}(r^*) = (\mathcal{L}(r)^* \cdot \mathcal{L}_{\text{weak}}(r)) \cup (\mathcal{L}(r))^\omega$$

$$\mathcal{L}_{\text{weak}}(r_1 \cup r_2) = \mathcal{L}_{\text{weak}}(r_1) \cup \mathcal{L}_{\text{weak}}(r_2)$$

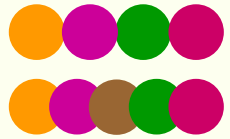
$$\mathcal{L}_{\text{weak}}(r_1 \cap r_2) = \mathcal{L}_{\text{weak}}(r_1) \cap \mathcal{L}_{\text{weak}}(r_2)$$



Bad fix 2: weak language

- Define

$$\begin{aligned} w \models r &\iff \\ &\exists j < |w| \text{ s.t. } w^{0..j} \in \mathcal{L}(r) \text{ or} \\ &r \in \mathcal{L}_{\text{weak}}(r) \end{aligned}$$



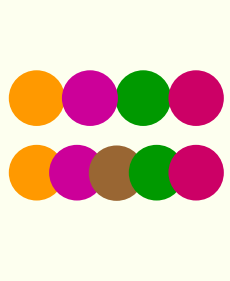
Now what's wrong?

- Another problem:
Should not have mixed the
"too short" & "too long" cases

Reason is related to
truncated semantics;
see paper for details

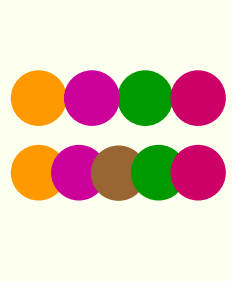
$\langle a \rangle$...
but $\langle c \rangle$ TT does not match r_2

$$\langle a \rangle \langle b \rangle \langle c \rangle \in \mathcal{L}_{\text{weak}}(r_1 \circ r_2)$$



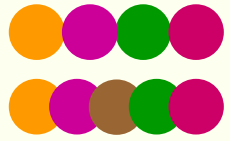
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Three languages of a SERE

- $\mathcal{L}(r)$ — as before
- $\mathcal{F}(r)$
 - Finite **proper** prefixes of words in $\mathcal{L}(r)$
 - Except: no opinion on satisfiability
 - $\mathcal{F}(a \cdot b \cdot c) = \mathcal{F}(a \cdot b \cdot \text{false}) = \mathcal{F}(a \cdot b \cdot (c \cap (c \cdot c)))$
- $\mathcal{I}(r)$
 - Infinite words in which we “get stuck” in a^*



Formal

No ω here, only words that are "too short"

$$\mathcal{F}(\lambda) = \emptyset$$

$$\mathcal{F}(b) = \varepsilon$$

Proper prefix!

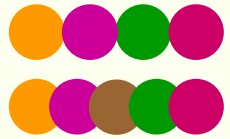
$$\mathcal{F}(r_1 \cdot r_2) = \mathcal{F}(r_1) \cup (\mathcal{L}(r_1) \cdot \mathcal{F}(r_2))$$

$$\mathcal{F}(r_1 \circ r_2) = \mathcal{F}(r_1) \cup (\mathcal{L}(r_1) \circ \mathcal{F}(r_2))$$

$$\mathcal{F}(r^*) = \mathcal{L}(r)^* \cdot \mathcal{F}(r)$$

$$\mathcal{F}(r_1 \cup r_2) = \mathcal{F}(r_1) \cup \mathcal{F}(r_2)$$

$$\mathcal{F}(r_1 \cap r_2) = \mathcal{F}(r_1) \cap \mathcal{F}(r_2)$$



Formally

Both empty - you can only get stuck in a *

$$\mathcal{I}(\lambda) = \emptyset$$

$$\mathcal{I}(b) = \emptyset$$

Here's the ω

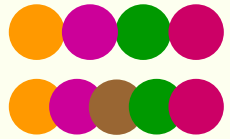
$$\mathcal{I}(r_1 \cdot r_2) = \mathcal{I}(r_1) \cup (\mathcal{L}(r_1) \cdot \mathcal{I}(r_2))$$

$$\mathcal{I}(r_1 \circ r_2) = \mathcal{I}(r_1) \cup (\mathcal{L}(r_1) \circ \mathcal{I}(r_2))$$

$$\mathcal{I}(r^*) = (\mathcal{L}(r)^* \cdot \mathcal{I}(r)) \cup (\mathcal{L}(r))^\omega$$

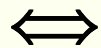
$$\mathcal{I}(r_1 \cup r_2) = \mathcal{I}(r_1) \cup \mathcal{I}(r_2)$$

$$\mathcal{I}(r_1 \cap r_2) = \mathcal{I}(r_1) \cap \mathcal{I}(r_2)$$



Semantics

- $w \models r$



$$\exists j > 0 \text{ s.t. } w^{0..j} \in \mathcal{L}(r) \text{ or } w \in \mathcal{I}(r) \cup \mathcal{F}(r) \cup \{\epsilon\}$$

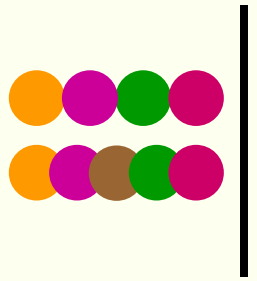
- By using proper prefixes, we avoid the problems of the bad fixes previously shown

ate $\mathcal{I}(\cdot)$ from $\mathcal{F}(\cdot)$

to play until we
ated semantics

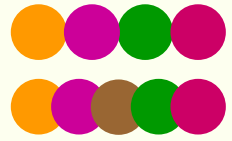
for details

on the empty
word



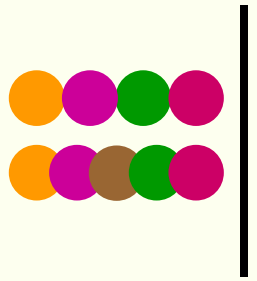
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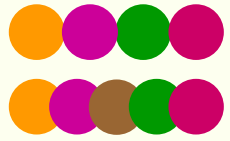
Good stuff about the fix

- Logical vs. structural treated consistently
- Preserves existing semantics
 - In the absence of structural contradictions
- Preserves properties of truncated semantics
 - ε models everything weakly, nothing strongly
 - Strength relation theorem
 - Prefix extension theorem
- Complexity is not affected



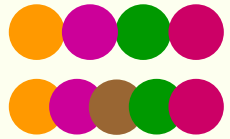
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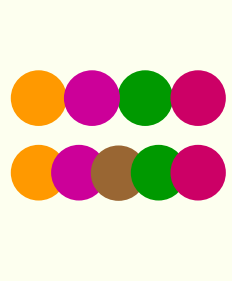
Back to political theory

- "We hold these truths to be self-evident, that **all men are created equal**"
 - Th. Jefferson, *Declaration of Independence*, 1776
- Surprisingly, this **does not hold** for **structural contradictions**



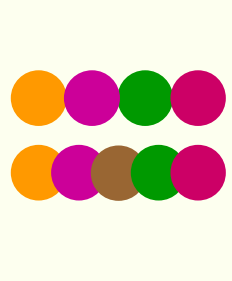
Not created equal

- Let $\varphi_1 = \text{true} \cap (\text{true} \cdot \text{true})$
 $\varphi_2 = (\text{true} \cdot \text{true}) \cap (\text{true} \cdot \text{true} \cdot \text{true})$
- Then on a word of length one
 - φ_1 does not hold
 - φ_2 does
- Note element of temporality
 - $\varphi_2 = \text{true} \cdot (\text{true} \cap (\text{true} \cdot \text{true}))$
 $= X \varphi_1$



Order of a formula

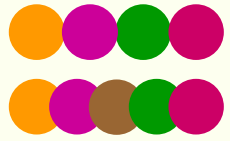
- Length of longest word w s.t. $w \models^- \varphi$
- Formulas of order 0:
 - false
 - $\text{true} \cap (\text{true} \cdot \text{true})$
- Formulas of order 1:
 - $\neg \text{false}$
 - $(\text{true} \cdot \text{true}) \cap (\text{true} \cdot \text{true} \cdot \text{true})$



Still not created equal!

- Let $\varphi_1 = (p \cdot p) \cap (p \cdot p \cdot p)$
 $\varphi_2 = (\text{true} \cdot \text{true}) \cap (\text{true} \cdot \text{true} \cdot \text{true})$
- Then on $w = \langle \rangle$
 - φ_1 does not hold
 - φ_2 does
- Although both are of order 1

One letter, no
propositions hold



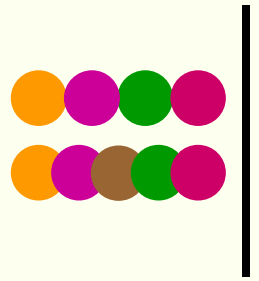
Logical vs. structural

$$\text{false} \equiv \text{true} \cap (\text{true} \cdot \text{true})$$

- And in general

$$X^{n-1} \text{ false} \equiv \text{true}^n \cap (\text{true}^n \cdot \text{true}^+)$$

- But what about $(\lambda \circ \text{true})$?
 - Doesn't it have a logical equivalent?
 - Yes, but there are subtleties
 - See paper for details



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Future work:

find a topological
characterization that works for
 \cap and \circ in new semantics

(Previously:

topological characterization
proving that without \cap and \circ ,
the relation between $r!$ and r is
the same as between U and W) $\mathcal{I}(\cdot)$

- Structures not created equal